בס"ד

First Greater Boston Math Olympiad
5th Grade

Your name: _____

Try to solve as many problems as you can, in any order you choose. There are six problems, and a correct solution of each of them wins you the number of points shown in parentheses. Show your work. If your answer is wrong but your method is correct, you will get partial credit. If necessary, use back sides of pages or attach additional sheets, putting your name on them.

Good luck!

*****PLEASE DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO*****

1. (6) Solve: INK + INK + INK + INK + INK + INK = PEN
(INK and PEN are 3-digit numbers, and different letters stand for different digits).

Answer:

You can use the space below for the scratch work.

2. Money in Wonderland comes in $5 and $7 bills.

(**a**) (4) What is the smallest amount of money you need to have in order to buy a slice of pizza which costs $1 and get back your change in full? (The pizza man has plenty of $5 and $7 bills.) For example, having $7 won't do, since the pizza man can only give you $5 back.

Answer:

Explanation:

(**b**) (8) Vending machines in Wonderland accept only exact payments (do not give back change). List all positive integer numbers which CANNOT be used as prices in such vending machines. (That is, find the sums of money that cannot be paid by exact change.)

Answer:

Explanation:

3. Two people play a game. They put 3 piles of matches on the table:
the first one contains 1 match, the second one 3 matches, and the third one 4 matches.
Then they take turns making moves. In a move, a player may take any nonzero number of matches FROM ONE PILE. The player who takes the last match from the table loses the game.

(**a**) (5) The player who makes the first move can win the game.
What is the winning first move?

Answer:

(**b**) (6) How can he win? (Describe his strategy.)

Answer:

4. (**a**) (4) How many times in a half-day (= 12 hours) the hour and the minute hand of a clock form the right angle with each other?

Answer:

Explanation:

(**b**) (8) How many times in a half-day the seconds hand of a clock falls on the line bisecting the angle between the hour and the minute hands?

Answer:

Explanation:

5. (**a**) (4) Put 5 points on the plane so that each 3 of them are vertices of an isosceles triangle (i.e., a triangle with two equal sides), and no three points lie on the same line.

(**b**) (8) Do the same with 6 points.

Answers:

You can use the space below for the scratch work.

6. (7) The number A2...2B has 2004 digits (all digits standing between A and B are 2). This number is divisible by 72. Find the digits A and B. (**Hint**: use the fact that $72 = 8 \times 9$. Find B first and then A.)

Answer:

Explanation: