GREATER BOSTON MATH OLYMPIAD 2006 GRADE 5

There are six problems, all weighted equally. Try to solve as many as you can, in any order you choose. Do not get discouraged if you cannot find a solution: many of these problems are difficult, though few require much calculation. Write your answers in the white paper forms. Explain why you think your answer is correct: merely guessing a correct answer will not receive full credit. Do not forget to sign on the top of the solution form (white paper). Good luck!

Problem 1. In this multiplication example, same letters mean same digits, different letters mean different digits, and stars mean arbitrary digits. Find A and B.

$$\begin{array}{r} ******* A \\ \qquad\qquad A \\ \hline BBBBBBBBB \end{array}$$

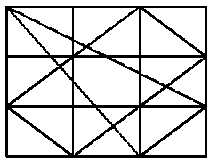
Problem 2. A row of 10 digits is written according to the following rule: the first three digits are chosen arbitrarily, and then each next digit is the last digit of the sum of the previous three. For example, starting with 1-2-3 yields 1-2-3-6-1-0-7-8-5-0. Which three digits should go first so that the *last* three are 7-7-7?

1	**2**	**3**	6	1	0	7	**8**	**5**	**0**

?	**?**	**?**					**7**	**7**	**7**

Problem 3. To pay his income tax, a pirate has to give 8 piles of golden coins, arranged in such a way that no two piles have same number of coins, and no two piles combined have same number of coins as a third pile. What is the minimal number of coins the pirate has to pay?

Problem 4. A rectangle is divided into 9 equal rectangles, and a parallelogram and a triangle are drawn inside as shown on the right. The perimeter of the parallelogram is 6 feet. The perimeter of the triangle is x feet. Find the largest integer which is smaller than x, and the smallest integer which is larger than x.



Problem 5. In Math-annapolis, chicken nuggets can be ordered in boxes of 4,6, and 17. What is the largest number such that you can not order any combination of the above to achieve exactly the number you want?

Problem 6. One wants to use tiles of form  and  to make a square without a unit size corner. This can be done when the square is 4-by-4 units, as shown on the right. Among the squares of dimensions 5-by-5, 6-by-6, 7-by-7, etc., up to 100-by-100, how many are those for which this can be done?